# Cell Library Characterization at Low Voltage using Non-linear Operating Point Analysis of Local Variations

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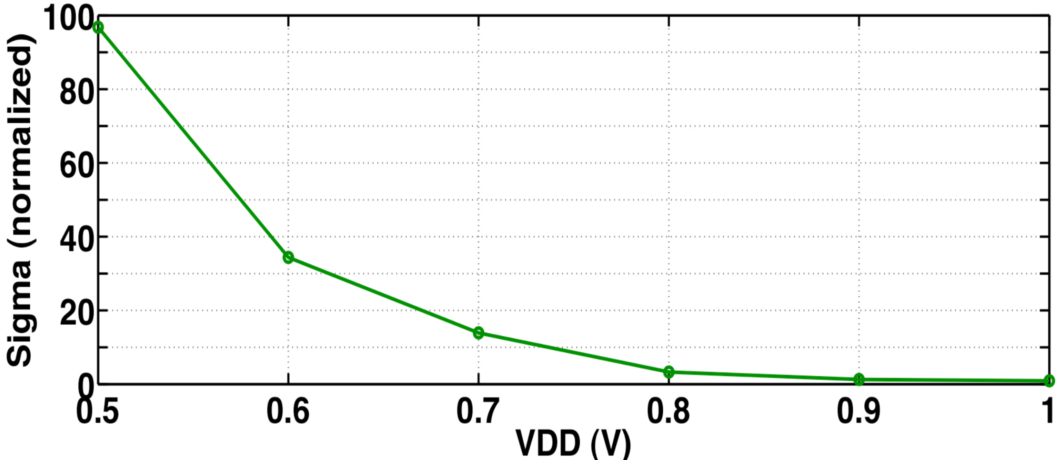
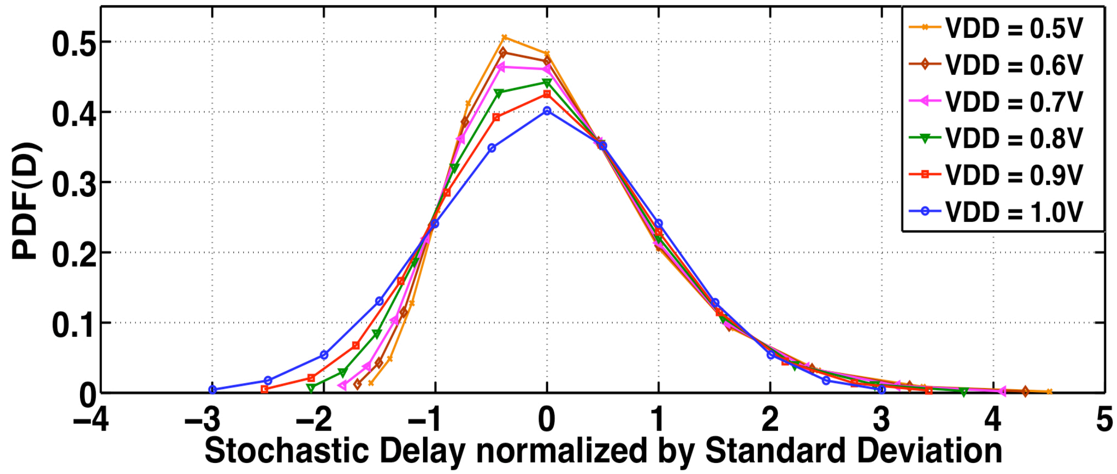
# Motivation

- For technology nodes of 65nm and below, local transistor variations become increasingly significant for logic timing
- Variation is much more significant at ULV ($V_{DD} \leq 0.5V$)



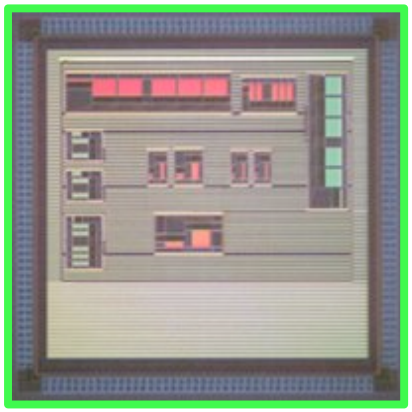

Standard deviation increases by 100X from 1V to 0.5V

Delay PDF becomes more non-Gaussian from 1V to 0.5V

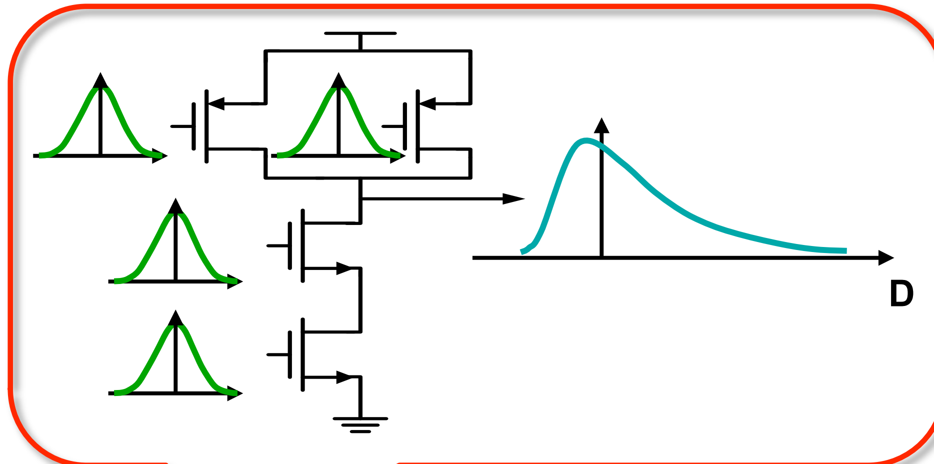


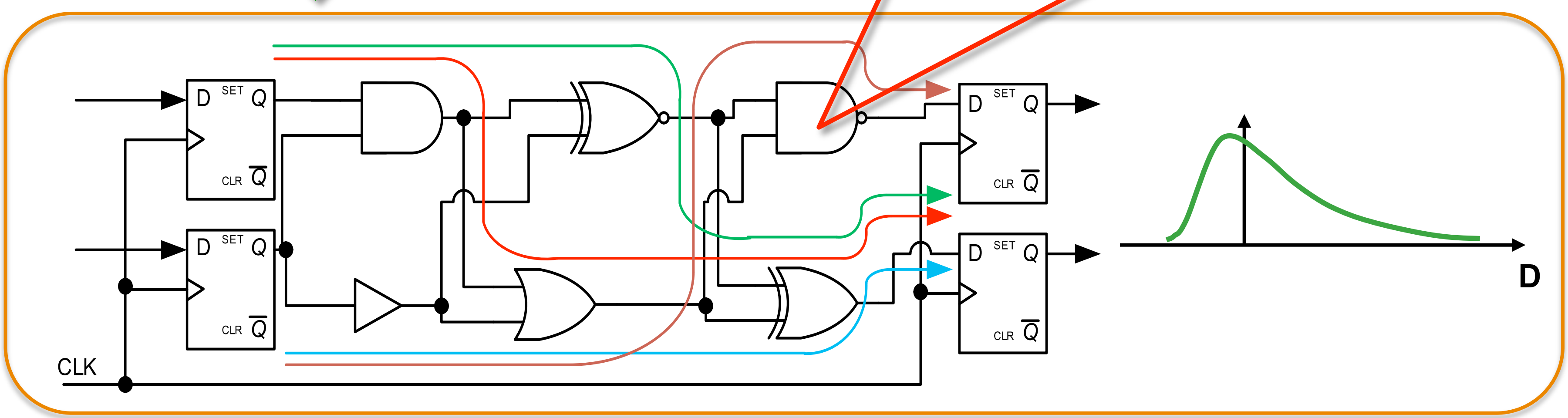
# Motivation

## Full-chip Timing Closure



## Cell Library Characterization



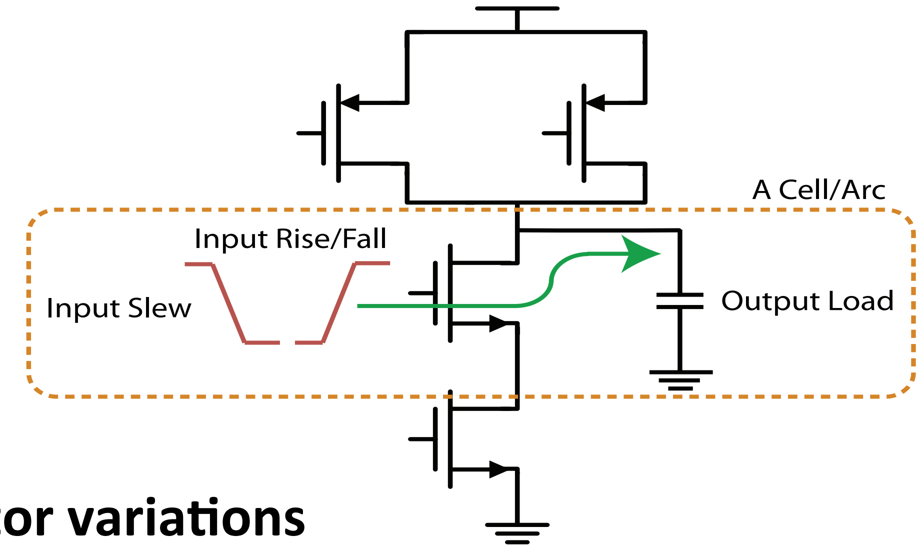

## Timing Path Analysis

# Motivation

- **Existing approaches:**
  - **Corner based analysis does not account for local variations**
  - **Monte-Carlo is accurate but computationally expensive**
  - **Gaussian approximation is computationally efficient but inaccurate**
- **NLOPALV approach:**
  - **It is highly accurate (within 5% compared to Monte-Carlo) at low voltage**
  - **It is computationally efficient**
  - **It is integrated in standard IC design flow using commercial CAD tools**

# Definitions

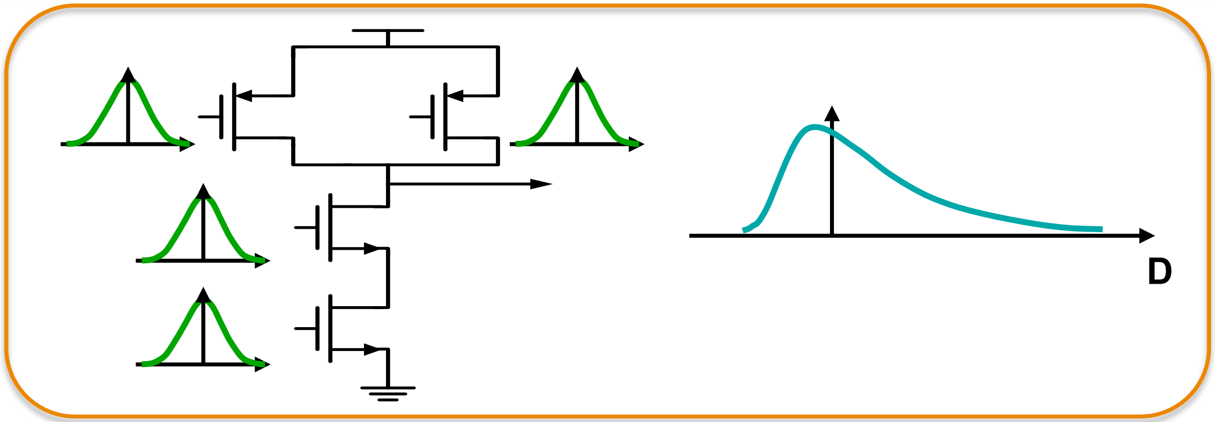
- **Cell/Arc is defined by:**
  - **Input trigger edge (rise/fall)**
  - **Input slew**
  - **Output capacitive load**




- **Stochastic Delay (D):**
  - **Resulting from local transistor variations**
  - **Stochastic variation in the cell delay relative to global corner delay**
- **Stochastic Output Slew (S):**
  - **Stochastic variation in output slew relative to corner output slew**
- **Cell Characterization Output:**
  - **PDF of stochastic delay for each cell/arc**
  - **Stochastic output slew corresponding to each stochastic delay**

# Approach

**Goal: Determine the PDF of the stochastic delay for each Cell/Arc**




**Generic Problem**

- **Determine the PDF of Stochastic delay D, where:**
  - **Stochastic delay D is a non-linear function of transistor random variables (RVs) $x_i$: $D = D(x_1, x_2, \ldots x_N)$**
  - **Transistor RVs $x_i$ generally have non-Gaussian PDFs**
  - **For this work, the $x_i$ are Gaussian**

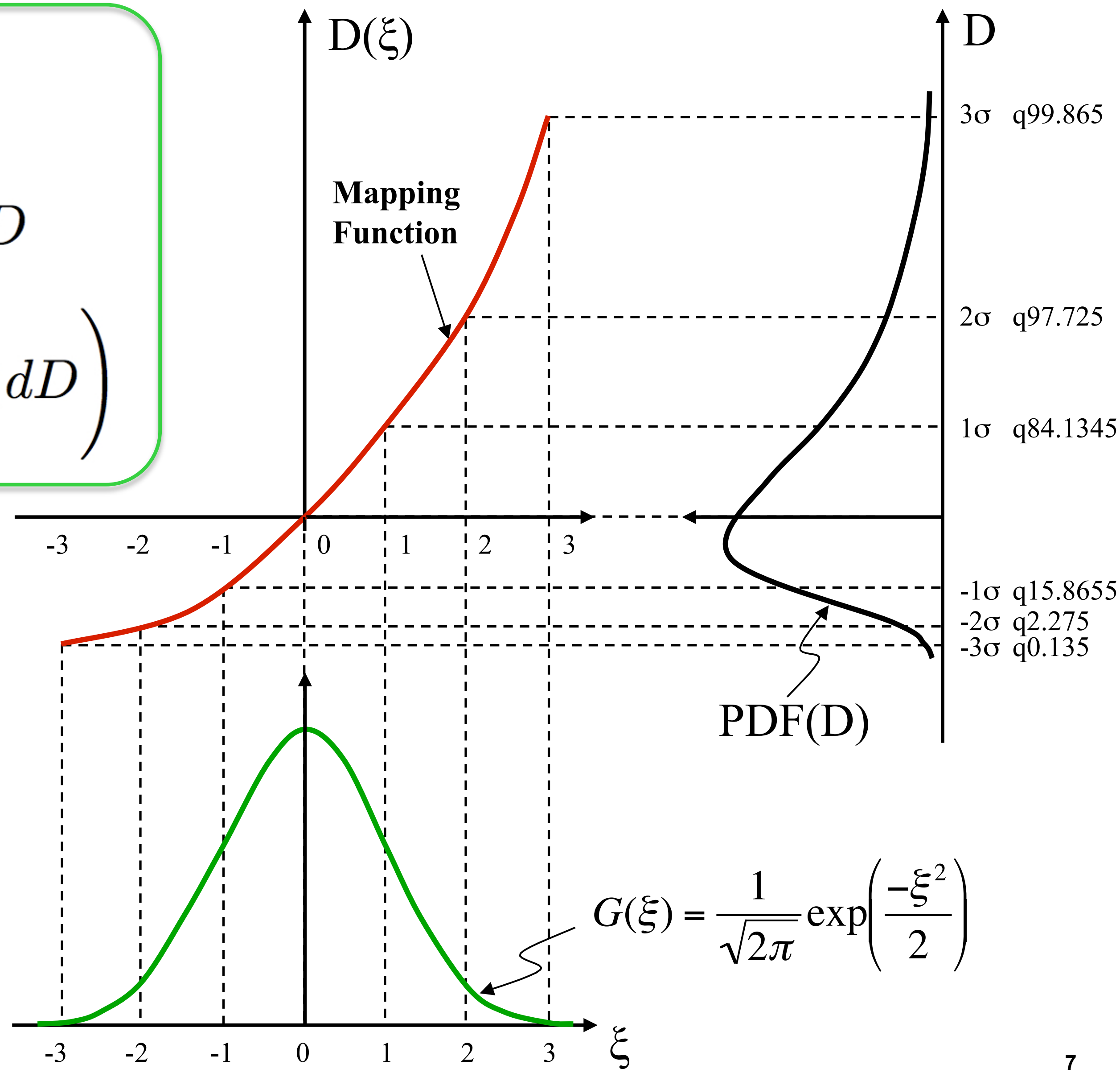
# Approach

$$\Phi(\xi) = \int_{-\infty}^{\xi} G(\xi) d\xi$$

$$\Phi(\xi) = \int_{-\infty}^{D} P(D) dD$$

$$\xi = \Phi^{-1}\left(\int_{-\infty}^{D} P(D) dD\right)$$

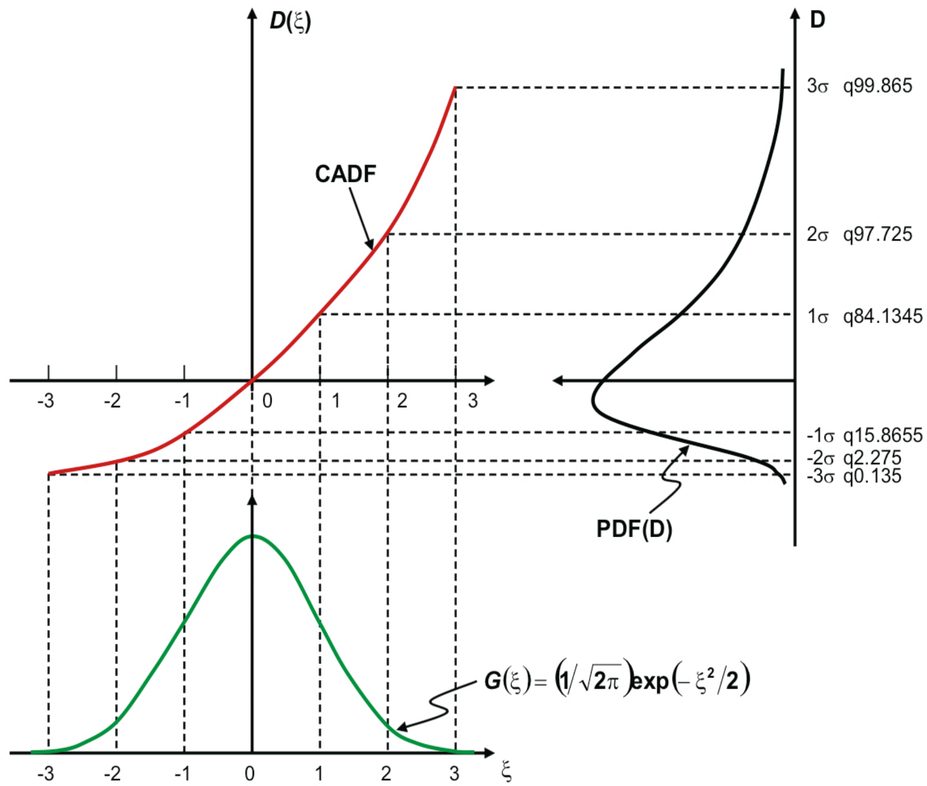
**Mapping non-Gaussian PDF on a standard normal variable**



# Approach

**Outputs of Cell Characterization:**

- **D(ξ) and S(ξ) for all cell/arcs in the library**
- **Characterization range: $-3 \leq \xi \leq 3$**




**CADF D(ξ) uniquely defines $P_D(D)$ and CASF S(ξ) uniquely defines the output slew**

**Library characterization in terms of D(ξ) and S(ξ) simplify the use of characterized cells in Timing Path Analysis**

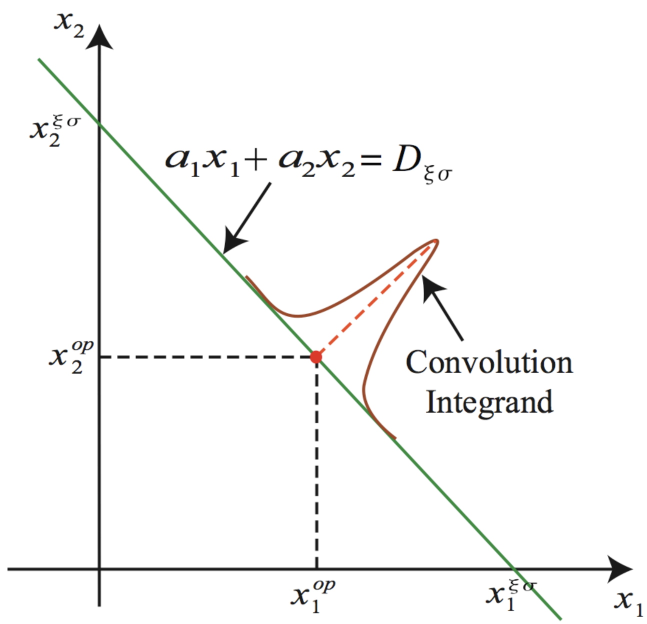
# NLOPALV Approach

- **For each value of ξ**
  - **Determine the ξ-sigma operating point**
  - **Evaluate the cell delay at that operating point using SPICE**
- **To visualize the importance of operating point, consider the Linear-Gaussian example**

**Cell delay PDF is convolution of RV PDFs**

$$P_D(D) = \int_{-\infty}^{\infty} P_{y_1}(y) P_{y_2}(D-y)\,dy$$

$$y_i = a_i x_i$$



# NLOPALV Approach

$$P_D(D) = \int_{-\infty}^{\infty} P_{y_1}(y) P_{y_2}(D-y)dy$$

$$= k_1 \exp\left(\frac{-D^2}{2(\alpha_1^2+\alpha_2^2)}\right)\int_{-\infty}^{\infty} \exp\left(\frac{-(\alpha_1^2+\alpha_2^2)(y-\frac{\alpha_1^2}{\alpha_1^2+\alpha_2^2}D)^2}{2\alpha_1^2\alpha_2^2}\right)dy$$

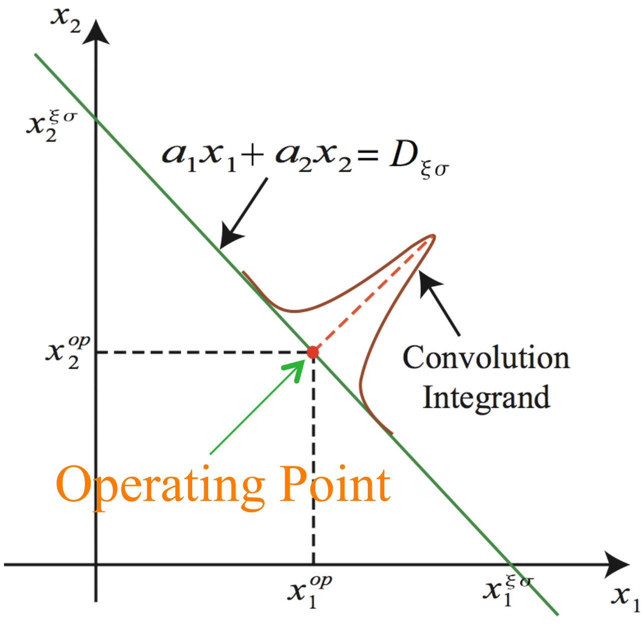
Gaussian with $\sigma_D = \sqrt{\alpha_1^2+\alpha_2^2}$

**Integrand of the convolution integral peaks at the operating point**

$$y_i^{op} = \frac{\alpha_i^2}{\alpha_1^2+\alpha_2^2} D_{\xi\sigma}$$

**In the space of standard normal variables:**

$$\zeta_i^{op} = \frac{\xi\alpha_i}{\sqrt{\alpha_1^2+\alpha_2^2}}$$



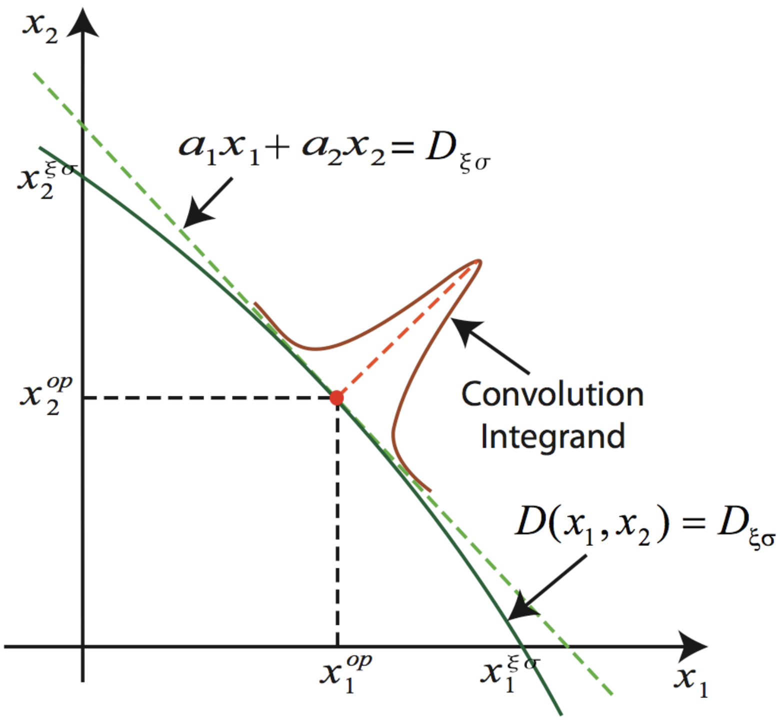
# NLOPALV Approach

**In the non-linear case:**

- **Non-linear delay curve can be linearized about the operating point**

**In the vicinity of the operating point:**

- **The linear approximation has small error**
- **Only points in the immediate vicinity of the operating point contribute significantly towards convolution**

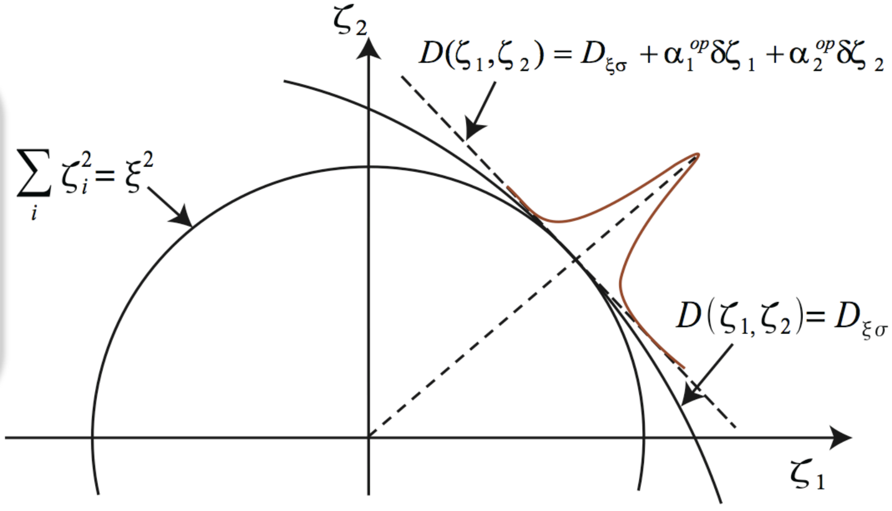



**This is the fundamental assumption underlying NLOPALV**

# Non-Linear – Non-Gaussian Case

$$D(\zeta_1, \zeta_2) = D_{\xi\sigma} + \alpha_1^{op}\delta\zeta_1 + \alpha_2^{op}\delta\zeta_2$$

$$\alpha_i^{op} = \left(\frac{dD}{d\zeta_i}\right)_{op} = \left(\frac{dD}{dx_i}\right)_{op}\left(\frac{dx_i}{d\zeta_i}\right)_{op}$$



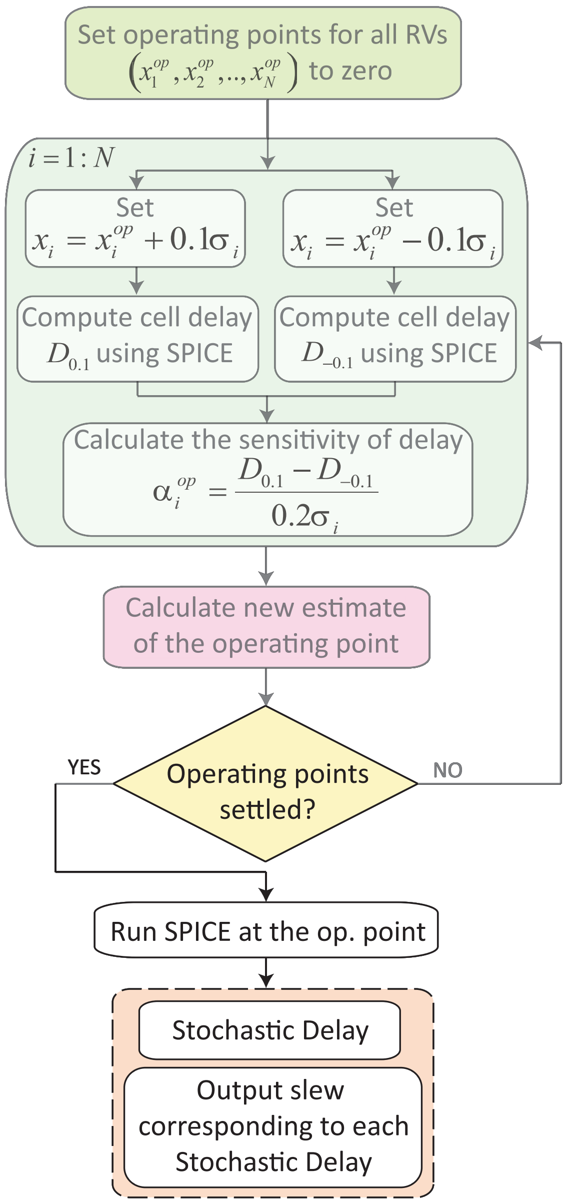

**The operating point is the point is determined iteratively as:**

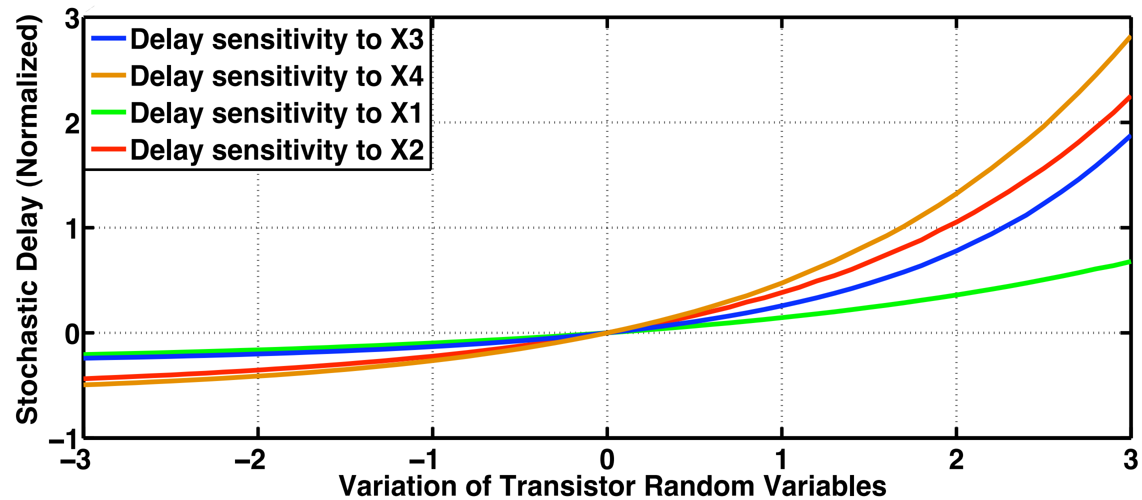
$$\zeta_i^{op} = \frac{\xi\alpha_i^{op}}{\sqrt{\sum_{j=1}^{N}(\alpha_j^{op})^2}}$$

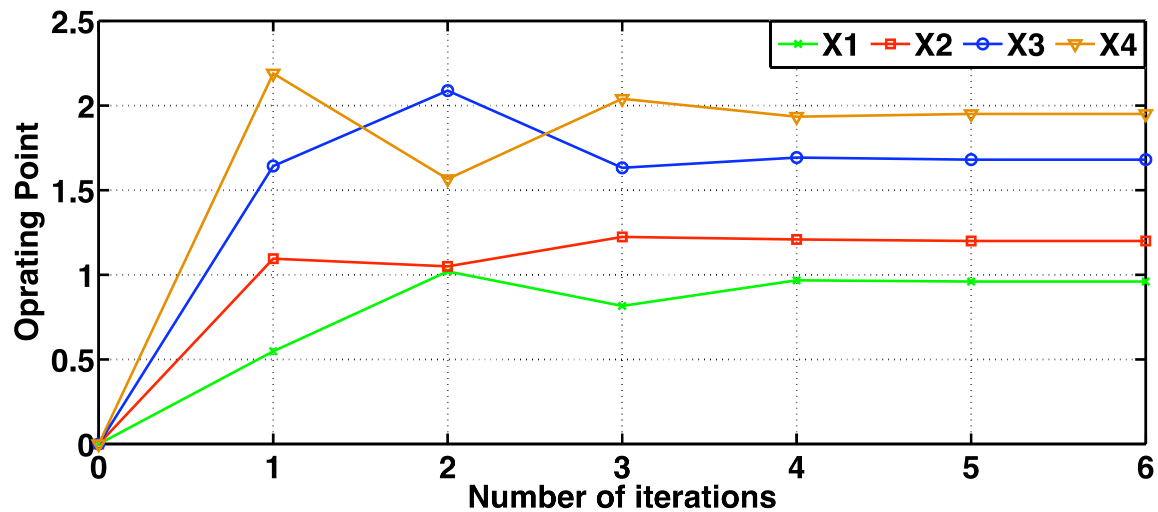
**Cell delay is computed as:**

$$D_{\xi\sigma} = D(\zeta_1^{op}, \zeta_2^{op}, \ldots \zeta_N^{op})$$

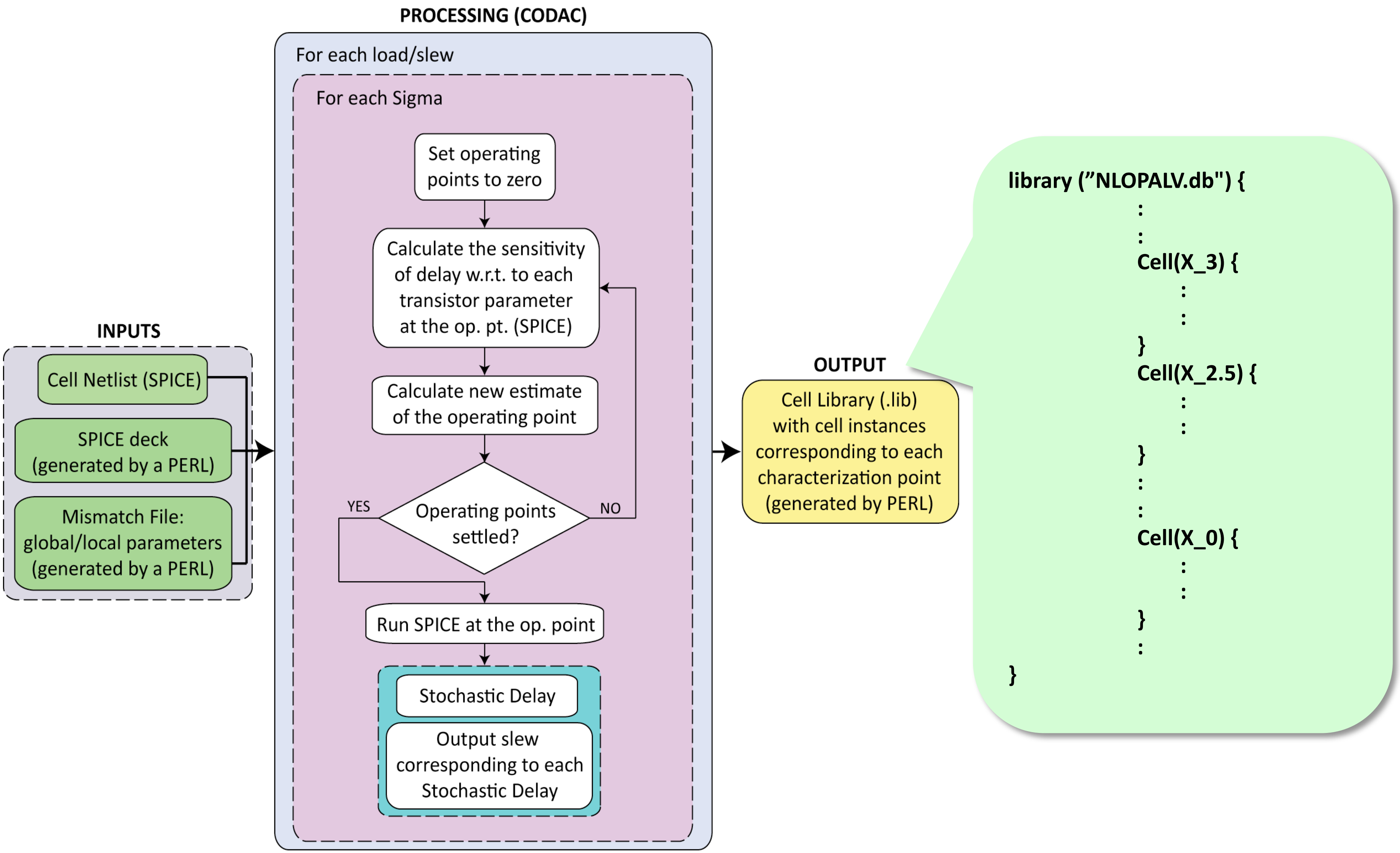
# Cell Characterization Flow



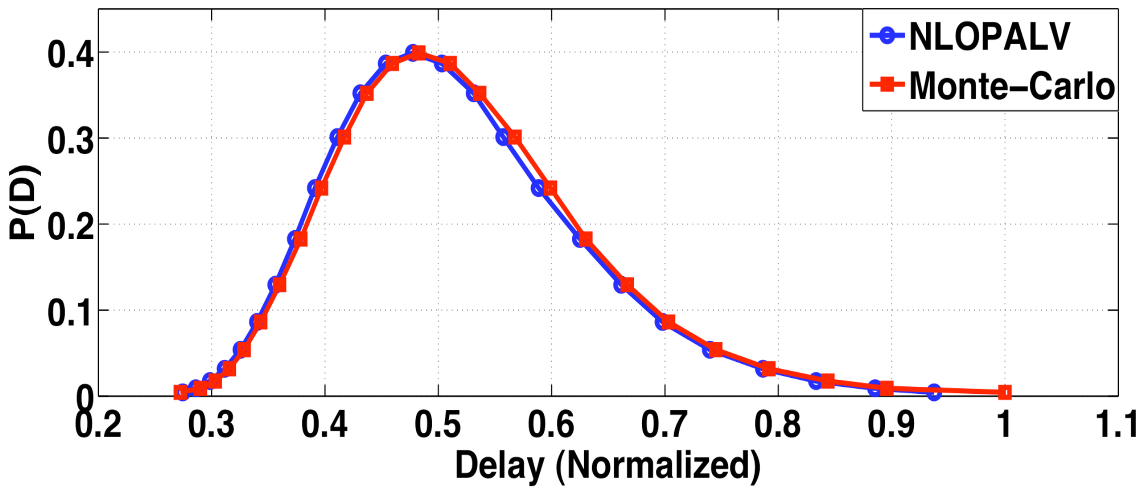



# Cell Characterization Flow

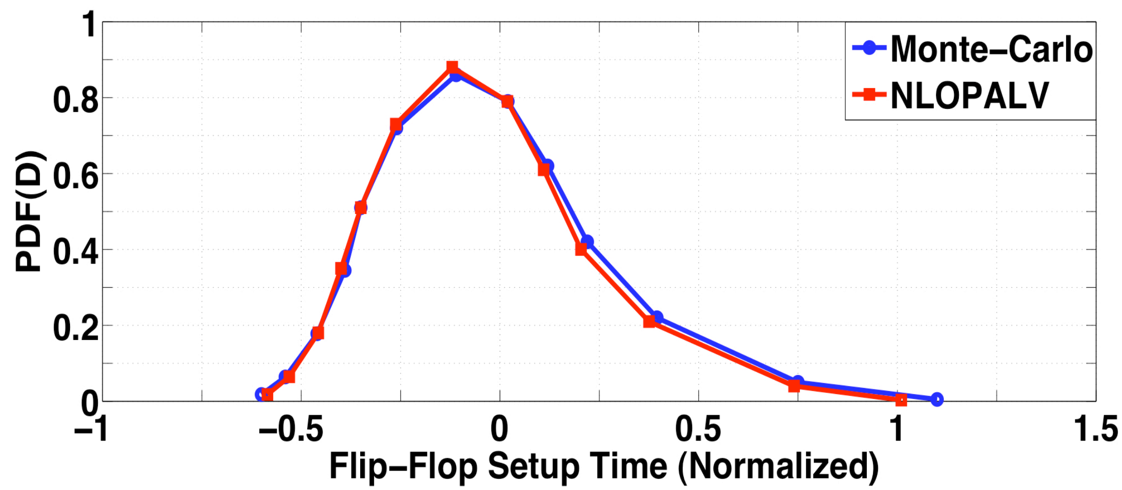


# Cell Characterization Results

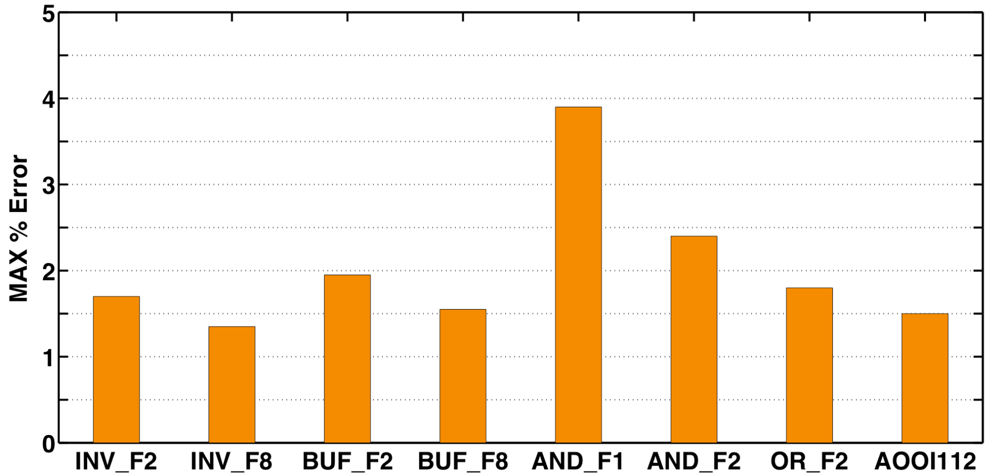



PDF of the total delay of an Adder cell

PDF of the stochastic delay of a Flip-Flop



# Cell Characterization Results

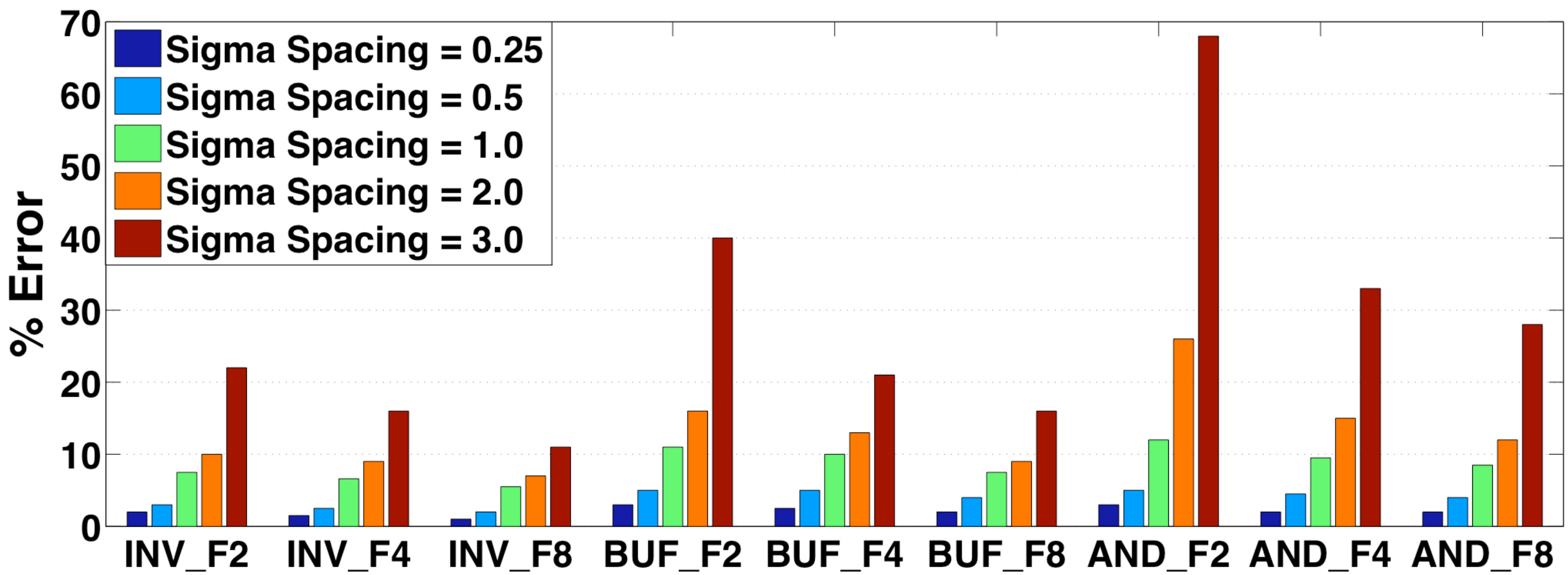



**Accuracy within 5% compared to Monte-Carlo**

| | | Monte-Carlo | NLOPALV | Gaussian Approx. |
|---|---|---|---|---|
| **NAND2** | SPICE Sims. | 10,000 | 70 | 20 |
| | % Error | 0% (Reference) | 5% | 90% |
| **Flip-Fop** | SPICE Sims. | 100,000 | 3,000 | 2,000 |
| | % Error | 0% (Reference) | 5% | 85% |

# Sigma Spacing vs. Accuracy

**Typical characterization range: -3 ≤ ξ ≤ 3**




**Sigma spacing of 0.5 can be used without much degradation in accuracy**

# Conclusions

- We have developed an approach to characterize stochastic delay of logic cells, which results from local variations
- The approach:
  - is computationally efficient
  - accurately predicts non-Gaussian delay PDFs
  - has been implemented using commercial STA tools
- A standard cell library of 130 cells, built using commercial 28nm CMOS technology, has been characterized for stochastic delay at low voltage
- Comparison with Monte-Carlo shows high accuracy of the approach

# Thank You!